Algebra of Vectors

1 Mark Questions

1. Find a vector in the direction of vector $2\hat{i} - 3\hat{j} + 6\hat{k}$ which has magnitude 21 units.

Foreign 2014

> To find a vector in the direction of given vector, first of all we find unit vector in the direction of given vector and then multiply it with given magnitude.

Let $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

$$|\vec{a}| = \sqrt{(2)^2 + (-3)^2 + (6)^2}$$
$$= \sqrt{4 + 9 + 36}$$
$$= \sqrt{49} = 7 \text{ units}$$ **(1/2)**

The unit vector in the direction of the given vector $\vec{a}$ is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{1}{7}(2\hat{i} - 3\hat{j} + 6\hat{k}) = \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$

Therefore, the vector of magnitude equal to 21 units and in the direction of $\vec{a}$ is

$$21\hat{a} = 21\left(\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right)$$
$$= 6\hat{i} - 9\hat{j} + 18\hat{k}$$ **(1/2)**

2. Find a vector $\vec{a}$ of magnitude $5\sqrt{2}$, making an angle of $\frac{\pi}{4}$ with X-axis, $\frac{\pi}{2}$ with Y-axis and an acute angle θ with Z-axis. **All India 2014**

Given, a vector $\vec{a}$ makes an angle $\frac{\pi}{4}$ with X-axis and $\frac{\pi}{2}$ with Y-axis.

So, $l = \cos\frac{\pi}{4}$ and $m = \cos\frac{\pi}{2} \Rightarrow l = \frac{1}{\sqrt{2}}, m = 0$

We know that, $l^2 + m^2 + n^2 = 1$

$$\Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + (0)^2 + n^2 = 1 \Rightarrow \frac{1}{2} + n^2 = 1$$

$$\Rightarrow n^2 = 1 - \frac{1}{2} \Rightarrow n = \pm\frac{1}{\sqrt{2}} \Rightarrow n = \frac{1}{\sqrt{2}}$$

$$\therefore \cos\theta = \frac{1}{\sqrt{2}}$$

[$\because \theta$ is an acute angle with Z-axis]

$$\Rightarrow \theta = \frac{\pi}{4}$$

Thus, direction cosines of a line are

$$\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$$ **(1/2)**

$\therefore$ Vector $\vec{a}$

$$= |\vec{a}|\left(\cos\frac{\pi}{4}\hat{i} + \cos\frac{\pi}{2}\hat{j} + \cos\frac{\pi}{4}\hat{k}\right)$$

$$= 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\hat{i} + (0)\hat{j} + \frac{1}{\sqrt{2}}\hat{k}\right) = 5\hat{i} + 5\hat{k}$$ **(1/2)**

3. Write a unit vector in the direction of the sum of the vectors $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

Delhi 2014C

Given, $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} - 7\hat{k}$

Now, $\vec{a} + \vec{b} = 2\hat{i} + 2\hat{j} - 5\hat{k} + 2\hat{i} + \hat{j} - 7\hat{k}$

$$= 4\hat{i} + 3\hat{j} - 12\hat{k}$$

and $|\vec{a} + \vec{b}| = \sqrt{(4)^2 + (3)^2 + (-12)^2}$

$$= \sqrt{16 + 9 + 144}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169} = 13 \text{ units}$$ **(1/2)**

$\therefore$ Required unit vector = Unit vector along the direction of $\vec{a} + \vec{b}$

$$= \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|} = \frac{4}{13}\hat{i} + \frac{3}{13}\hat{j} - \frac{12}{13}\hat{k}$$ **(1/2)**

4. Find the value of p for which the vectors $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are parallel.

All India 2014

We have, $3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\hat{i} - 2p\hat{j} + 3\hat{k}$ are two parallel vectors, so their direction ratios will be proportional.

$$\therefore \quad \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3} \Rightarrow \frac{2}{-2p} = \frac{3}{1}$$

$$\Rightarrow -6p = 2 \Rightarrow p = \frac{2}{-6} \Rightarrow p = -\frac{1}{3}$$ **(1)**

5. Write the value of cosine of the angle which the vector $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ makes with Y-axis.

Delhi 2014C

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of $\vec{a}$ is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{(1)^2 + (1)^2 + (1)^2}}$$

$$= \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

$\therefore$ Cosine of angle which given vector makes with Y-axis is $\frac{1}{\sqrt{3}}$. **(1)**

6. Find the angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$. **All India 2014C**

Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$

Now, unit vector in the direction of $\vec{a}$ is

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{1^2 + 1^2 + 1^2}} \Rightarrow \hat{a} = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$$

$$\Rightarrow \quad \hat{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

So, angle between X-axis and the vector $\hat{i} + \hat{j} + \hat{k}$ is $\cos\alpha = \frac{1}{\sqrt{3}}$

$$\Rightarrow \quad \alpha = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

[$\because \hat{a} = l\hat{i} + m\hat{j} + n\hat{k}$ and $\cos\alpha = l \Rightarrow \alpha = \cos^{-1} l$]

(1)

7. Write a vector in the direction of the vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 9 units.

Delhi 2014C

Let $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$

Comparing with $X = x\hat{i} + y\hat{j} + z\hat{k}$, we get

$$x = 1, y = -2, z = 2$$

Magnitude $|\vec{a}| = \sqrt{x^2 + y^2 + z^2}$

$$= \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = 3 \text{ units}$$

$\therefore$ Unit vector in the direction of given vector

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$$ **(1/2)**

Hence, the vector in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ which has magnitude 9 units is given by

$$9\hat{a} = 9\left(\frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}\right) = 3\hat{i} - 6\hat{j} + 6\hat{k}$$ **(1/2)**

8. Write a unit vector in the direction of vector $\overrightarrow{PQ}$, where $\vec{P}$ and $\vec{Q}$ are the points (1, 3, 0) and (4, 5, 6) respectively. **Foreign 2014**

Firstly, find the vector $\overrightarrow{PQ}$ by using the formula $(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$,

then required unit vector is given by $\dfrac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}$.

The given points are $\vec{P}(1, 3, 0)$ and $\vec{Q}(4, 5, 6)$.

Here, $x_1 = 1, y_1 = 3, z_1 = 0$

and $x_2 = 4, y_2 = 5, z_2 = 6$

So, vector $\overrightarrow{PQ}$

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$= (4 - 1)\hat{i} + (5 - 3)\hat{j} + (6 - 0)\hat{k}$$

$$= 3\hat{i} + 2\hat{j} + 6\hat{k}$$ **(1/2)**

$\therefore$ Magnitude of given vector

$$|\overrightarrow{PQ}| = \sqrt{3^2 + 2^2 + 6^2}$$

$$= \sqrt{9 + 4 + 36}$$

$$= \sqrt{49} = 7 \text{ units}$$

Hence, the unit vector in the direction of $\overrightarrow{PQ}$

$$\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|} = \frac{3\hat{i} + 2\hat{j} + 6\hat{k}}{7} = \frac{3}{7}\hat{i} + \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$$ **(1/2)**

9. Write the value of the following:

$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$ **Foreign 2014**

$\hat{i} \times (\hat{j} + \hat{k}) + \hat{j} \times (\hat{k} + \hat{i}) + \hat{k} \times (\hat{i} + \hat{j})$

$= \hat{i} \times \hat{j} + \hat{i} \times \hat{k} + \hat{j} \times \hat{k} + \hat{j} \times \hat{i} + \hat{k} \times \hat{i} + \hat{k} \times \hat{j}$

$= \hat{k} - \hat{j} + \hat{i} - \hat{k} + \hat{j} - \hat{i} = 0$

$[\because \hat{i} \times \hat{j} = \hat{k}, \hat{i} \times \hat{k} - \hat{j}, \hat{j} \times \hat{k} = \hat{i}, \hat{j} \times \hat{i} = -\hat{k},$
$\hat{k} \times \hat{i} = \hat{j}, \hat{k} \times \hat{j} = -\hat{i}]$ **(1)**

10. Write a unit vector in the direction of the sum of vectors $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = -\hat{i} + \hat{j} + 3\hat{k}$.

Delhi 2013

Let $\vec{c} = \vec{a} + \vec{b} = (2\hat{i} - \hat{j} + 2\hat{k}) + (-\hat{i} + \hat{j} + 3\hat{k})$

$= \hat{i} + 0\hat{j} + 5\hat{k}$

Now, $|\vec{c}| = |\vec{a} + \vec{b}| = \sqrt{1^2 + 5^2} = \sqrt{26}$

$\therefore \quad c = \dfrac{(\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} \quad \left[\because \hat{a} = \dfrac{\vec{a}}{|\vec{a}|}\right]$ **(1/2)**

$= \dfrac{1}{\sqrt{26}}\hat{i} + \dfrac{5}{\sqrt{26}}\hat{k}$ **(1/2)**

which is the required unit vector.

11. If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ and $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ are two equal vectors, then write the value of $x + y + z$.

HOTS; Delhi 2013

Two vectors are equal, if coefficients of their components are equal. **(1/2)**

Given, $\vec{a} = \vec{b}$

$\Rightarrow \quad x\hat{i} + 2\hat{j} - z\hat{k} = 3\hat{i} - y\hat{j} + \hat{k}$

$\therefore \quad x = 3, y = -2, z = -1$

Now, $x + y + z = 3 - 2 - 1 = 0$ **(1/2)**

12. P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$, respectively. Write the position vector of a point R which divides the line segment PQ in the ratio 2 : 1 externally.

All India 2013

Given, P and Q are two points with position vectors $3\vec{a} - 2\vec{b}$ and $\vec{a} + \vec{b}$ respectively. Also, point R divides the line segment PQ in the ratio 2 : 1 externally.

$\therefore$ Position vector of a point R

$$= \frac{2 \times (\vec{a} + \vec{b}) - 1 \times (3\vec{a} - 2\vec{b})}{2 - 1}$$

[by section formula]

$$= \frac{2\vec{a} + 2\vec{b} - 3\vec{a} + 2\vec{b}}{2 - 1}$$

$$= \frac{-\vec{a} + 4\vec{b}}{1} = -\vec{a} + 4\vec{b}$$ **(1)**

13. L and M are two points with position vectors $2\vec{a} - \vec{b}$ and $\vec{a} + 2\vec{b}$, respectively. Write the position vector of a point N which divides the line segment LM in the ratio 2 : 1 externally.

All India 2013

Given, L and M are two points with position vectors $2\vec{a}-\vec{b}$ and $\vec{a}+2\vec{b}$, respectively. Also, point N divides the line segment LM in the ratio 2:1 externally.

$\therefore$ Position vector of a point N

$$=\frac{2\times(\vec{a}+2\vec{b})-1\times(2\vec{a}-\vec{b})}{2-1}$$

[by section formula]

$$=\frac{2\vec{a}+4\vec{b}-2\vec{a}+\vec{b}}{1}=5\vec{b}$$ **(1)**

14. A and B are two points with position vectors $2\vec{a}-3\vec{b}$ and $6\vec{b}-\vec{a}$, respectively. Write the position vector of a point P which divides the line segment AB internally in the ratio 1 : 2.

All India 2013

Given, A and B are two points with position vectors $2\vec{a}-3\vec{b}$ and $6\vec{b}-\vec{a}$, respectively. Also, point P divides the line segment AB in the ratio 1 : 2 internally.

$\therefore$ Position vector of a point P

$$=\frac{1\times(6\vec{b}-\vec{a})+2\times(2\vec{a}-3\vec{b})}{1+2}$$

[by section formula]

$$=\frac{6\vec{b}-\vec{a}+4\vec{a}-6\vec{b}}{3}=\frac{3\vec{a}}{3}=\vec{a}$$ **(1)**

15. Find the sum of the vectors $\vec{a}=\hat{i}-2\hat{j}+\hat{k}$, $\vec{b}=-2\hat{i}+4\hat{j}+5\hat{k}$ and $\vec{c}=\hat{i}-6\hat{j}-7\hat{k}$.

Delhi 2012

Given vectors are $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$,

$\vec{b} = -2\hat{i} + 4\hat{j} + 5\hat{k}$ and $\vec{c} = \hat{i} - 6\hat{j} + 7\hat{k}$

Sum of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ is

$$\vec{a} + \vec{b} + \vec{c} = (\hat{i} - 2\hat{j} + \hat{k}) + (-2\hat{i} + 4\hat{j} + 5\hat{k}) + (\hat{i} - 6\hat{j} - 7\hat{k})$$

$$= -4\hat{j} - \hat{k} \quad \textbf{(1)}$$

16. Find the scalar components of $\overrightarrow{AB}$ with initial point $A(2, 1)$ and terminal point $B(-5, 7)$.

HOTS ; All India 2012

> Scalar components of $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$
> = Position vector of $\vec{B}$ – Position vector of $\vec{A}$.

Given, initial point is $A(2, 1)$ and terminal point is $B(-5, 7)$ then scalar component of $\overrightarrow{AB}$ are $x_2 - x_1$ and $y_2 - y_1$.

$\Rightarrow \quad -5 - 2 = -7$ and $7 - 1 = 6$ **(1)**

17. For what values of $\vec{a}$, the vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $a\hat{i} + 6\hat{j} - 8\hat{k}$ are collinear?

HOTS ; Delhi 2011

If $\vec{a}$ and $\vec{b}$ are collinear, then $\vec{a} = \pm\lambda\vec{b}$ or $|\vec{a}| = \lambda \cdot |\vec{b}|$

Let given vectors are $\vec{a} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ and $\vec{b} = a\hat{i} - 6\hat{j} - 8\hat{k}$

Vectors $\vec{a}$ and $\vec{b}$ are said to be collinear, if

$$\vec{a} = k \cdot \vec{b}, \text{ where } k \text{ is a scalar}$$

$\therefore \quad 2\hat{i} - 3\hat{j} + 4\hat{k} = k(a\hat{i} + 6\hat{j} - 8\hat{k})$

Above equation is satisfied, when $a = -4$.

$\therefore \quad a = -4$ **(1)**

18. Write the direction cosines of vector $-2\hat{i} + \hat{j} - 5\hat{k}$. **Delhi 2011**

Direction cosines of the vector $a\hat{i} + b\hat{j} + c\hat{k}$ are

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

Let $\vec{a} = -2\hat{i} + \hat{j} - 5\hat{k}$

$\therefore$ Direction cosines of $\vec{a}$ are

$$\frac{-2}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}, \frac{1}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$$

and $$\frac{-5}{\sqrt{(-2)^2 + (1)^2 + (-5)^2}}$$

i.e. $$\frac{-2}{\sqrt{30}}, \frac{1}{\sqrt{30}}, \frac{-5}{\sqrt{30}}$$ **(1)**

19. Write the position vector of mid-point of the vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$. **Foreign 2011**

Mid-point of the position vectors
$\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and

$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ is $\dfrac{\vec{a} + \vec{b}}{2}$

or $\dfrac{(a_1 + b_1)\hat{i} + (a_2 + b_2)\hat{j} + (a_3 + b_3)\hat{k}}{2}$

Given, points are $P(2, 3, 4)$ and $Q(4, 1, -2)$ whose position vectors $\overrightarrow{OP} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\overrightarrow{OQ} = 4\hat{i} + \hat{j} - 2\hat{k}$.

Now, position vector of mid-point of vector joining points $P(2, 3, 4)$ and $Q(4, 1, -2)$ is

$$\overrightarrow{OR} = \frac{\overrightarrow{OP} + \overrightarrow{OQ}}{2} = \frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) + (4\hat{i} + \hat{j} - 2\hat{k})}{2}$$

$$\therefore \overrightarrow{OR} = \frac{6\hat{i} + 4\hat{j} + 2\hat{k}}{2} = 3\hat{i} + 2\hat{j} + \hat{k}$$ **(1)**

20. Write a unit vector in the direction of vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$. **Delhi 2009; All India 2011**

We know that, unit vector in the direction of $\vec{a}$ is $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$\therefore$ Required unit vector in the direction of vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k} = \dfrac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{(2)^2 + (1)^2 + (2)^2}}$

$$= \frac{2\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$ **(1)**

21. Find the magnitude of the vector $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$. **All India 2011C; Delhi 2008**

Magnitude of a vector $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is $|\vec{r}| = \sqrt{x^2 + y^2 + z^2}$.

Given, vector is $\vec{a} = 3\hat{i} - 2\hat{j} + 6\hat{k}$

Magnitude of $\vec{a} = |\vec{a}| = \sqrt{(3)^2 + (-2)^2 + (6)^2}$

$= \sqrt{9 + 4 + 36} = \sqrt{49} = 7$ units **(1)**

22. Find a unit vector in the direction of vector $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. **All India 2011C**

Given vector is $\vec{a} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

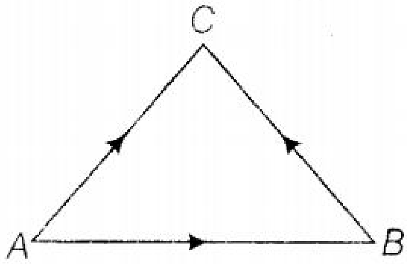
Unit vector in the direction of $\vec{a} = \dfrac{\vec{a}}{|\vec{a}|}$

$$= \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (3)^2 + (6)^2}} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} + \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$ **(1)**

23. If A, B and C are the vertices of a ΔABC, then what is the value of $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA}$? **Delhi 2011C**

Let ΔABC be the given triangle.



Now, by triangle law of vector addition, we have $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\Rightarrow \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} + \overrightarrow{AC}$

[adding $\overrightarrow{CA}$ on both sides]

$\Rightarrow \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{CA} - \overrightarrow{CA}$ [$\because \overrightarrow{AC} = -\overrightarrow{CA}$]

$\Rightarrow \quad \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \vec{0}$ **(1)**

24. Find a unit vector in the direction of $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$. **Delhi 2011C**

Given vector is $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$

Unit vector in the direction of $\vec{a}$

$$= \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{(2)^2 + (-3)^2 + (6)^2}} = \frac{2\hat{i} - 3\hat{j} + 6\hat{k}}{\sqrt{49}}$$

$$= \frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}$$ **(1)**

25. Find a vector in the direction of $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$, which has magnitude 6 units. **Delhi 2010C**

Do same as Que. 7. [**Ans.** $4\hat{i} - 2\hat{j} + 4\hat{k}$]

26. Find a position vector of mid-point of the line segment AB, where A is point (3, 4, −2) and B is point (1, 2, 4). **Delhi 2010**

Do same as Que. 19. [**Ans.** $2\hat{i} - 3\hat{j} + \hat{k}$]

27. Write a vector of magnitude 9 units in the direction of vector $-2\hat{i}+\hat{j}+2\hat{k}$. **All India 2010**

Do same as Que. 7. [**Ans.** $-6\hat{i}-3\hat{j}+6\hat{k}$]

28. Write a vector of magnitude 15 units in the direction of vector $\hat{i}-2\hat{j}+2\hat{k}$. **Delhi 2010**

Do same as Que. 7. [**Ans.** $5\hat{i}-10\hat{j}+10\hat{k}$]

29. What is the cosine of angle which the vector $\sqrt{2}\hat{i}+\hat{j}+\hat{k}$ makes with Y-axis? **HOTS; Delhi 2010**

Let $\vec{a}=\sqrt{2}\hat{i}+\hat{j}+\hat{k}$

Now, unit vector in the direction of $\vec{a}$ is

$$\hat{a}=\frac{\vec{a}}{|\vec{a}|}=\frac{\sqrt{2}\hat{i}+\hat{j}+\hat{k}}{\sqrt{(\sqrt{2})^2+(1)^2+(1)^2}}=\frac{\sqrt{2}\hat{i}+\hat{j}+\hat{k}}{2}$$

$$=\frac{\sqrt{2}}{2}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}$$

∴ Cosine of angle which given vector makes with Y-axis is 1/2. **(1)**

Alternate Method

The direction cosines along Y-axis are $(0, 1, 0)$, then we use the following cosine formula

$$\because \quad \theta=\cos^{-1}\{l_1l_2+m_1m_2+n_1n_2\}$$

$$\Rightarrow \quad \cos\theta=l_1l_2+m_1m_2+n_1n_2$$

Let $\vec{a}=\sqrt{2}\hat{i}+\hat{j}+\hat{k}$

Its direction cosines form

$$\vec{a}=\frac{\sqrt{2}}{2}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}=\frac{1}{\sqrt{2}}\hat{i}+\frac{1}{2}\hat{j}+\frac{1}{2}\hat{k}$$

[∵ Direction cosine of vector $a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$$=\frac{a_1}{\sqrt{a_1^2+a_2^2+a_3^2}}, \frac{a_2}{\sqrt{a_1^2+a_2^2+a_3^2}},$$

$\sqrt{a_1^2 + a_2^2 + a_3^2}$ $\sqrt{a_1^2 + a_2^2 + a_3^2}$

and $\left.\frac{a_3}{\sqrt{a_1^2 + a_2^2 + a_3^2}}\right]$

and let the position along with Y-axis is

$$\vec{b} = 0\hat{i} + 1\hat{j} + 0\hat{k}$$

If θ is the angle between them.

$\therefore$ $\cos\theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

$$\Rightarrow \qquad \theta = \cos^{-1}\left\{\frac{1}{\sqrt{2}}\cdot 0 + \frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0\right\}$$

$$\therefore \qquad \cos\theta = \frac{1}{2}$$

So, cosine of angle which the given vector makes with Y-axis is $\frac{1}{2}$. **(1)**

30. Find a unit vector in the direction of vector $\vec{b} = 6\hat{i} - 2\hat{j} + 3\hat{k}$. **All India 2009C**

Do same as Que. 24. $\left[\textbf{Ans. } \frac{6}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{3}{7}\hat{k}\right]$

31. Find a unit vector in the direction of vector $\vec{a} = -2\hat{i} + \hat{j} + 2\hat{k}$. **Delhi 2009C**

Do same as Que. 24. $\left[\textbf{Ans. } \frac{-2}{3}\hat{i} + \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right]$

32. Write a unit vector in the direction of $\vec{a} = 2\hat{i} - 6\hat{j} + 3\hat{k}$. **Delhi 2009**

Do same as Que. 24. $\left[\textbf{Ans. } \frac{2}{7}\hat{i} - \frac{6}{7}\hat{j} + \frac{3}{7}\hat{k}\right]$

33. Find the magnitude of the vector $\vec{a} = 2\hat{i} - 6\hat{j} - 3\hat{k}$. **All India 2008C**

Given, vector is $\vec{a} = 2\hat{j} - 6\hat{j} - 3\hat{k}$

Magnitude of $\vec{a} = |\vec{a}| = \sqrt{(2)^2 + (-6)^2 + (-3)^2}$

$= \sqrt{4 + 36 + 9} = \sqrt{49} = 7$ units **(1)**

34. Find a unit vector in the direction of $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$. **Delhi 2008C**

Do same as Que. 24.

$$\left[\textbf{Ans. } \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}\right]$$

35. If $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 5\hat{k}$, then find a unit vector in the direction $\vec{a} - \vec{b}$.
All India 2008

Firstly, find $\vec{a} - \vec{b}$ by subtracting the components $\hat{i}$, $\hat{j}$ and $\hat{k}$ of $\vec{a}$ and $\vec{b}$ simultaneously and then use the formula $= \dfrac{\vec{a} - \vec{b}}{|\vec{a} - \vec{b}|}$ to get required unit vector.

Given, $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$ and $3\hat{i} + \hat{j} - 5\hat{k}$

$$\therefore \quad \vec{a} - \vec{b} = (\hat{i} + 2\hat{j} - \hat{k}) - (3\hat{i} + \hat{j} - 5\hat{k})$$
$$= -2\hat{i} + \hat{j} + 4\hat{k}$$

Let $\vec{a} - \vec{b} = \vec{c} = -2\hat{i} + \hat{j} + 4\hat{k}$

Now, unit vector in the direction of $\vec{c}$ is

$$\hat{c} = \frac{\vec{c}}{|\vec{c}|} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{(-2)^2 + (1)^2 + (4)^2}} = \frac{-2\hat{i} + \hat{j} + 4\hat{k}}{\sqrt{21}}$$

$$= \frac{-2}{\sqrt{21}}\hat{i} + \frac{1}{\sqrt{21}}\hat{k} + \frac{4}{\sqrt{21}}\hat{k} \quad \textbf{(1)}$$

which is the required unit vector.

36. Find a vector of magnitude 5 units and parallel to the resultant of $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$. **HOTS ; Delhi 2011**

Firstly, find resultant of the vectors $\vec{a}$ and $\vec{b}$, which is $\vec{a}+\vec{b}$. Then, find a unit vector in the direction of $\vec{a}+\vec{b}$ i.e. the unit vector multiplying by 5.

Given $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

Now, resultant of above vectors $= \vec{a} + \vec{b}$

$= (2\hat{i} + 3\hat{j} - \hat{k}) + (\hat{i} - 2\hat{j} + \hat{k}) = 3\hat{i} + \hat{j}$ **(1)**

Let $\vec{a} + \vec{b} = \vec{c}$

$\therefore \quad \vec{c} = 3\hat{i} + \hat{j}$

Now, unit vector $\hat{c}$ in the direction of $\vec{c}$, i.e.

$$= \frac{\vec{c}}{|\vec{c}|}$$ **(1)**

$$= \frac{3\hat{i} + \hat{j}}{\sqrt{(3)^2 + (1)^2}} = \frac{3\hat{i} + \hat{j}}{\sqrt{10}} = \frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}$$ **(1)**

Hence, vector of magnitude 5 units and parallel to resultant of $\vec{a}$ and $\vec{b}$ is

$$5\left(\frac{3}{\sqrt{10}}\hat{i} + \frac{1}{\sqrt{10}}\hat{j}\right) \text{ or } \frac{15}{\sqrt{10}}\hat{i} + \frac{5}{\sqrt{10}}\hat{j}$$ **(1)**

37. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$. Find a vector of magnitude 6 units, which is parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$. **All India 2010**

Firstly, find the vector $2\vec{a} - \vec{b} + 3\vec{c}$, then find the vector in the direction of $2\vec{a} - \vec{b} + 3\vec{c}$, i.e. the unit vector multiplying by 6.

Given, $\vec{a} = \hat{i} + \hat{j} + \hat{k}; \vec{b} = 4\hat{i} - 2\hat{j} + 3\hat{k}$

and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$\therefore \quad 2\vec{a} - \vec{b} + 3\vec{c}$

$= 2(\hat{i} + \hat{j} + \hat{k}) - (4\hat{i} - 2\hat{j} + 3\hat{k}) + 3(\hat{i} - 2\hat{j} + \hat{k})$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 4\hat{i} + 2\hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$\Rightarrow 2\vec{a} - \vec{b} + 3\vec{c} = \hat{i} - 2\hat{j} + 2\hat{k}$ **(1½)**

Now, a unit vector in the direction of vector

$$2\vec{a} - \vec{b} + 3\vec{c} = \frac{2\vec{a} - \vec{b} + 3\vec{c}}{|2\vec{a} - \vec{b} + 3\vec{c}|}.$$

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(1)^2 + (-2)^2 + (2)^2}} = \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{9}}$$

$$= \frac{\hat{i} - 2\hat{j} + 2\hat{k}}{3} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$ **(1½)**

Hence, vector of magnitude 6 units parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c} = 6\left(\frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

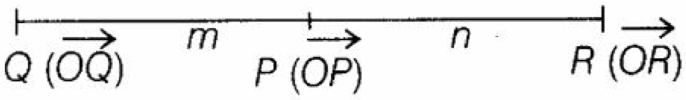
$= 2\hat{i} - 4\hat{j} + 4\hat{k}$ **(1)**

38. Find the position vector of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segment RQ.

HOTS; Delhi 2010

Here, we use the section formula for external division

$$\overrightarrow{OR} = \frac{m(\overrightarrow{OQ}) - n(\overrightarrow{OP})}{m-n}$$



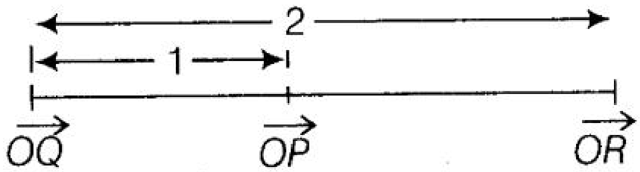
and then use the mid-point formula

$$\overrightarrow{OP} = \frac{\overrightarrow{OQ} + \overrightarrow{OR}}{2}$$

Given, $\overrightarrow{OP}$ = Position vector of $P = 2\vec{a} + \vec{b}$

and $\overrightarrow{OQ}$ = Position vector of $Q = \vec{a} - 3\vec{b}$

Let $\overrightarrow{OR}$ be the position vector of point R, which divides PQ in the ratio 1 : 2 externally



$$\therefore \quad \overrightarrow{OR} = \frac{1(\vec{a} - 3\vec{b}) - 2(2\vec{a} + \vec{b})}{1-2}$$

$$\left[\because \overrightarrow{OR} = \frac{m(\overrightarrow{OQ}) - n(\overrightarrow{OP})}{m-n} \cdot \text{Here, } m = 1, n = 2\right]$$

(1)

$$= \frac{\vec{a} - 3\vec{b} - 4\vec{a} - 2\vec{b}}{-1} = \frac{-3\vec{a} - 5\vec{b}}{-1} = 3\vec{a} + 5\vec{b}$$

Hence, $\overrightarrow{OR} = 3\vec{a} + 5\vec{b}$ **(1½)**

Now, we have to show that P is the mid-point of RQ.

i.e. $\overrightarrow{OP} = \dfrac{\overrightarrow{OR} + \overrightarrow{OQ}}{2}$

We have, $\overrightarrow{OR} = 3\vec{a} + 5\vec{b}$, $\overrightarrow{OQ} = \vec{a} - 3\vec{b}$

$$\therefore \quad \frac{\overrightarrow{OR} + \overrightarrow{OQ}}{2} = \frac{(3\vec{a} + 5\vec{b}) + (\vec{a} - 3\vec{b})}{2}$$

$$= \frac{4\vec{a} + 2\vec{b}}{2}$$

$$= \frac{2(2\vec{a} + \vec{b})}{2} = 2\vec{a} + \vec{b}$$

$$= \overrightarrow{OP} \qquad [\because \overrightarrow{OP} = 2\vec{a} + \vec{b}, \text{ given}]$$ **(1½)**

Hence, P is the mid-point of line segment R.